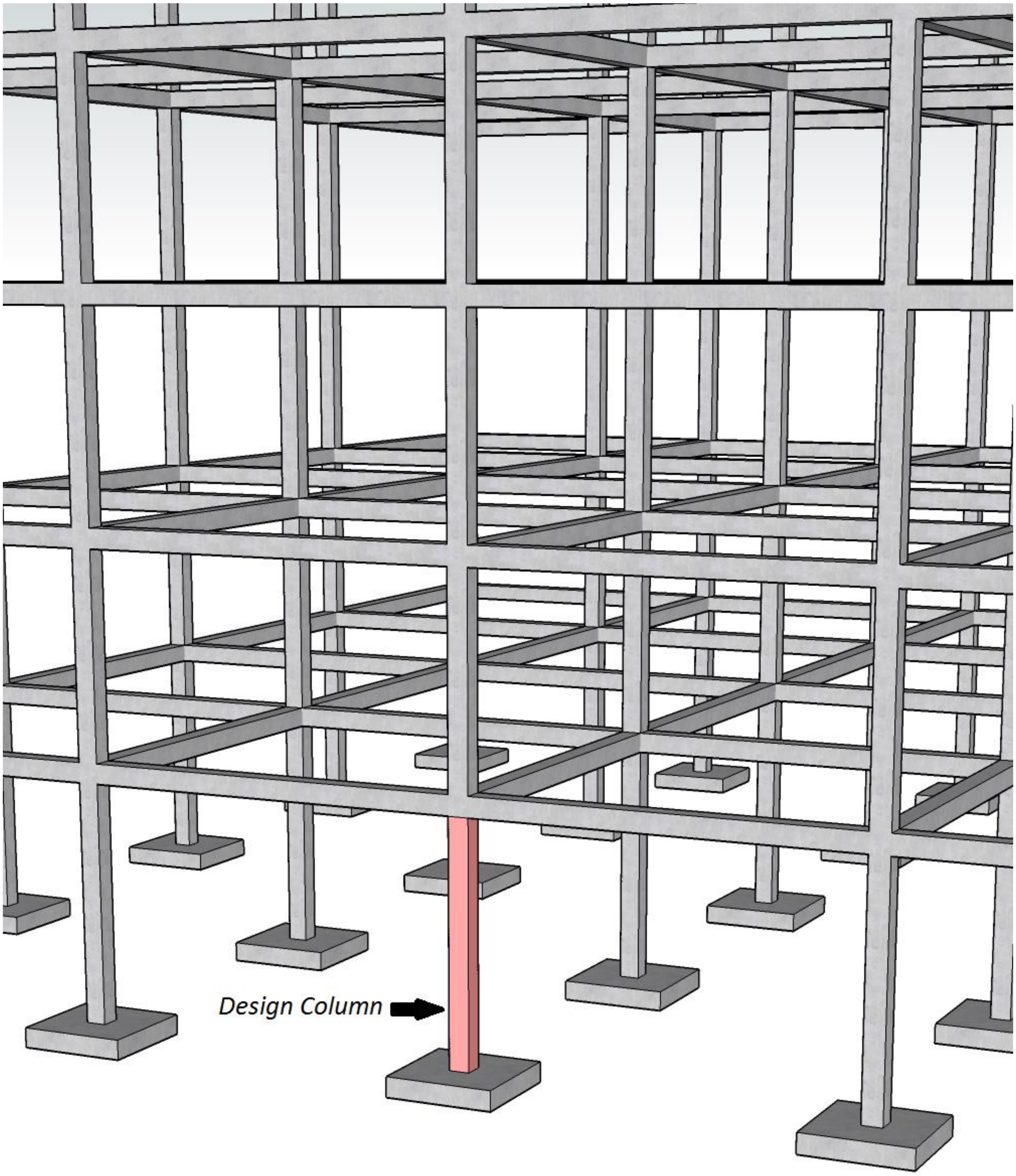
Slenderness Effects for Columns in Non-Sway Frame - Moment Magnification Method (ACI 318-14)



Slender Concrete Column Design in Non-Sway Frame Buildings

Evaluate slenderness effects for columns in a non-sway multistory reinforced concrete frame by determining the adequacy of the square tied column shown below, which is an exterior first floor column. The design forces obtained from a first-order analysis are provided in the design data section below. The story height is 12 ft. it is assumed that the frame is braced sufficiently to prevent relative translation of its joints. Assume 40% of the factored axial load is sustained. Compare the calculated results with the values presented in the Reference and with exact values from spColumn engineering software program from StructurePoint.

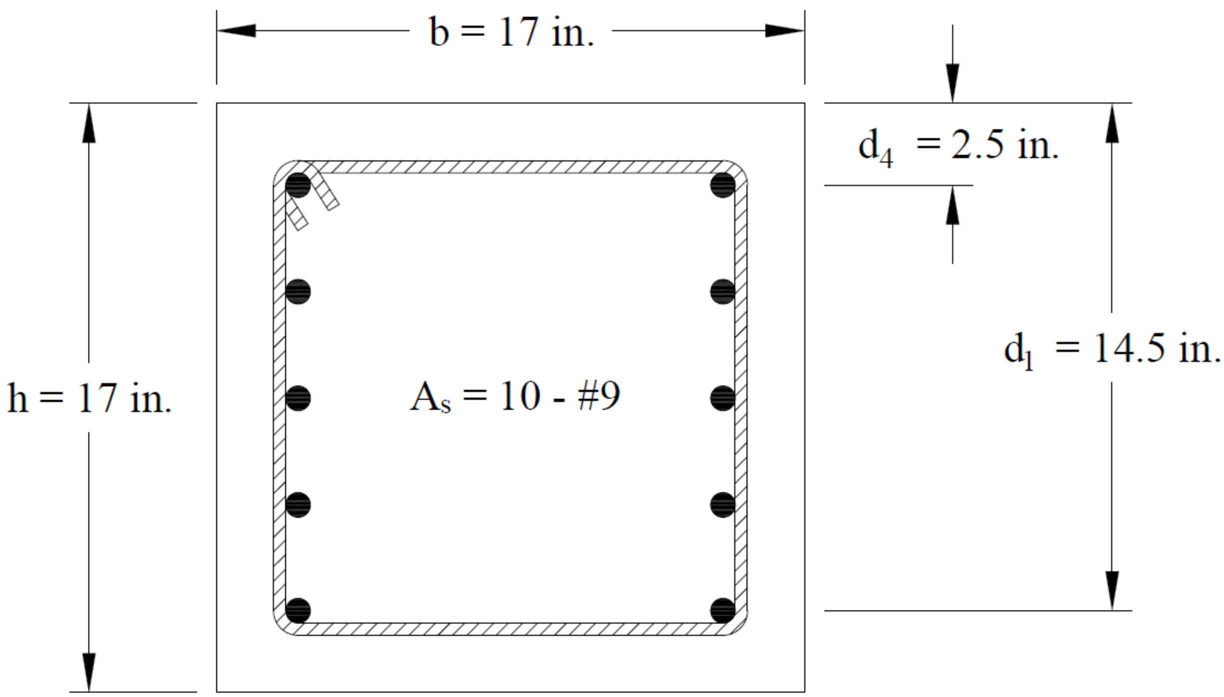


Figure 1 – Reinforced Concrete Column Cross-Section

Contents

Code

Building Code Requirements for Structural Concrete (ACI 318-14) and Commentary (ACI 318R-14)

Reference

Reinforced Concrete Design, Eighth Edition, 2018, Wang C. et. al., Example 13.17.3.

Design Data

Concrete f_c' = 3000 psi

Steel f_y = 60000 psi

Beams: h = 24 in., b = 14 in., l = 7 m

Columns: h = 17 in., b = 17 in. H = 12 ft

factored design forces obtained from first-order analysis from the reference:

Table 1 - Column factored loads			
Load Case	Axial Load, kip	Bending Moment, kip.ft	
		Top	Bottom
Factored Load	1776*	105	0
* Assume 40% of the axial load is sustained			

1. Slenderness Effects and Sway or Non-sway Frame Designation

Columns and stories in structures are considered as non-sway frames if the stability index for the story (Q) does not exceed 0.05. ***ACI 318-14 (6.6.4.3)***

The reference assumed that the frame is a non-sway frame since Q value is less than 0.05.

2. Determine Slenderness Effects

The reference decided to be consistent with the more conservative procedure provided by **ACI 318-14 (6.6.4.4.3)** by taking k value equals to 1.0. However, the k value, in this example, is calculated based on the exact procedure for illustration purposes.

$$I_{column} = 0.7 \times \frac{c^4}{12} = 0.7 \times \frac{17^4}{12} = 4872 \text{ in.}^4$$ ***ACI 318-14 (6.6.3.1.1(a))***

$$I_{beam} = 0.35 \times \frac{b \times h^3}{12} = 0.35 \times \frac{14 \times 24^3}{12} = 5645 \text{ in.}^4$$ ***ACI 318-14 (6.6.3.1.1(a))***

$$E = 57,000 \times \sqrt{f_c^{'}} = 57,000 \times \sqrt{3000} = 3122 \text{ ksi}$$ ***ACI 318-14 (19.2.2.1.b)***

For columns:

$$\frac{E \times I_{column}}{l_c} = \frac{3122 \times 4872}{12 \times 12} = 8.8 \times 10^3 \text{ kip.ft}$$

For beams framing into the columns:

$$\frac{E \times I_{beam}}{l_b} = \frac{3122 \times 5645}{30 \times 12} = 4.08 \times 10^3 \text{ kip.ft}$$

$$\Psi_A = \frac{\left(\sum \frac{EI}{l_c}\right)_{columns}}{\left(\sum \frac{EI}{l}\right)_{beams}} = \frac{2 \times 8.80 \times 10^3}{4.08 \times 10^3} = 4.32$$ ***ACI 318-14 (Figure R6.2.5)***

$\Psi_B = \infty$ (Column was assumed hinged at base) ***ACI 318-14 (Figure R6.2.5)***

Using Figure R6.2.5 from ACI 318-14 → k = 0.959 as shown in the figure below for the exterior columns with one beam framing into them in the directions of analysis.

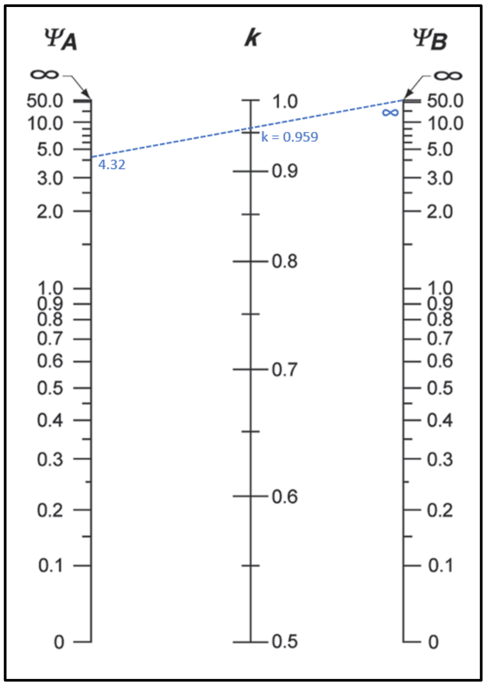


Figure 2 – Effective Length Factor (*k*) (Non-Sway Frame)

ACI 318-14 allows to neglect the slenderness in a non-sway frame if:

$$\frac{k \times l_u}{r} \leq 34 + 12\left(\frac{M_1}{M_2}\right)$$ ***ACI 318-14 (6.2.5b)***

Where:

$$r = \text{radius of gyration} = (a)\ \sqrt{\frac{I_g}{A_g}} \quad or \quad \text{(b) } 0.3 \times c_1$$ ***ACI 318-14 (6.2.5.1)***

$$r = \sqrt{\frac{I_g}{A_g}} = \sqrt{\frac{c^2}{12}} = \sqrt{\frac{17^2}{12}} = 4.91 \text{ in.}$$

$$\frac{0.959 \times (12 \times 12 - 24)}{4.91} = 23.45 < 34 - 12\left(\frac{0}{105}\right) = 34 \qquad \therefore \text{ slenderness can be neglected.}$$

Even though it is not required to consider slenderness effects for this column, the moment magnification method will be shown for illustration.

3. Moment Magnification – Non-Sway Frame

$M_{c2} = \delta M_2$ **_ACI 318-14 (6.6.4.5.1)_**

Where:

$$\delta = \text{ magnification factor} = \frac{C_m}{1 - \dfrac{P_u}{0.75 P_c}} \geq 1.0$$ **_ACI 318-14 (6.6.4.5.2)_**

$$P_c = \frac{\pi^2 (EI)_{eff}}{(kl_u)^2}$$ **_ACI 318-14 (6.6.4.4.2)_**

Where:

$$(EI)_{eff} = \begin{cases} \text{(a)} & \dfrac{0.4 E_c I_g}{1 + \beta_{dns}} \\ \text{(b)} & \dfrac{0.2 E_c I_g + E_s I_{se}}{1 + \beta_{dns}} \\ \text{(c)} & \dfrac{E_c I}{1 + \beta_{dns}} \end{cases}$$ **_ACI 318-14 (6.6.4.4.4)_**

There are three options for calculating the effective flexural stiffness of slender concrete columns $(EI)_{eff}$. The second equation provides accurate representation of the reinforcement in the section and will be used in this example and is also used by the solver in spColumn. Further comparison of the available options is provided in "Effective Flexural Stiffness for Critical Buckling Load of Concrete Columns" technical note.

3.1. Calculation of Critical Load (P_c)

$$I_{column} = \frac{c^4}{12} = \frac{17^4}{12} = 6960 \text{ in.}^4$$ **_ACI 318-14 (Table 6.6.3.1.1(a))_**

$$E_c = 57{,}000 \times \sqrt{f_c'} = 57{,}000 \times \sqrt{3000} = 3122 \text{ ksi}$$ **_ACI 318-14 (19.2.2.1.a)_**

β_{dns} is the ratio of maximum factored sustained axial load to maximum factored axial load associated with the same load combination. **_ACI 318-14 (6.6.4.4.4)_**

In this example, it is assumed that 40% of the factored axial load is sustained.

$$\beta_{dns} = \frac{P_{u,sustained}}{P_u} = \frac{0.4 \times P_u}{P_u} = 0.40 < 1.00 \rightarrow \therefore \beta_{dns} = 0.40$$

With 10-#9 reinforcement equally distributed on two sides and 17 in. x 17 in. column section → I_{se} = 360 in.^{4}.

$$(EI)_{eff} = \frac{0.2 E_c I_g + E_s I_{se}}{1 + \beta_{dns}}$$ **_ACI 318-14 (6.6.4.4.4(b))_**

$$(EI)_{eff} = \frac{0.2\times 3122\times 6960+29,000\times 360}{1+0.4} = 10.56\times 10^{6}\ \text{kip-in.}^{2}$$

$$P_c = \frac{\pi^2\times 10.56\times 10^6}{\left(0.959\times(12-2)\times 12\right)^2} = 7871\ \text{kip}$$

3.2. Calculation of Magnified Moment (M_c)

$$C_m = 0.6+0.4\frac{M_1}{M_2}$$ ***ACI 318-14 (6.6.4.5.3a)***

$$C_m = 0.6+0.4\left(\frac{0}{105}\right) = 0.6$$

$$\delta = \frac{C_m}{1-\dfrac{P_u}{0.75P_c}} \geq 1.0$$ ***ACI 318-14 (6.6.4.5.2)***

$$\delta = \frac{0.6}{1-\dfrac{525}{0.75\times 7871}} = 0.66 < 1.00 \rightarrow \ \delta = 1.00$$

$$M_{\min} = P_u\left(0.6+0.03h\right)$$ ***ACI 318-14 (6.6.4.5.4)***

Where P_u = 525 kip, and h = the section dimension in the direction being considered = 17 in.

$$M_{\min} = 525\left(\frac{0.6+0.03\times 17}{12}\right) = 48.56\ \text{kip.ft}$$

$$M_2 = 105\ \text{kip.ft} > M_{2,\min} = 48.56\ \text{kip.ft} \rightarrow M_2 = 105\ \text{kip.ft}$$ ***ACI 318-14 (6.6.4.5.4)***

$$M_{c2} = \delta M_2$$ ***ACI 318-14 (6.6.4.5.1)***

$$M_{c2} = 1.00\times 105 = 105\ \text{kip.ft}$$

4. Column Design

Based on the factored axial loads and magnified moments considering slenderness effects, the capacity of the assumed column section (17 in. × 17 in. with 10 – #9 bars distributed on two sides) will be checked and confirmed to finalize the design. A column interaction diagram will be generated using strain compatibility analysis, the detailed procedure to develop column interaction diagram can be found in "Interaction Diagram - Tied Reinforced Concrete Column" example.

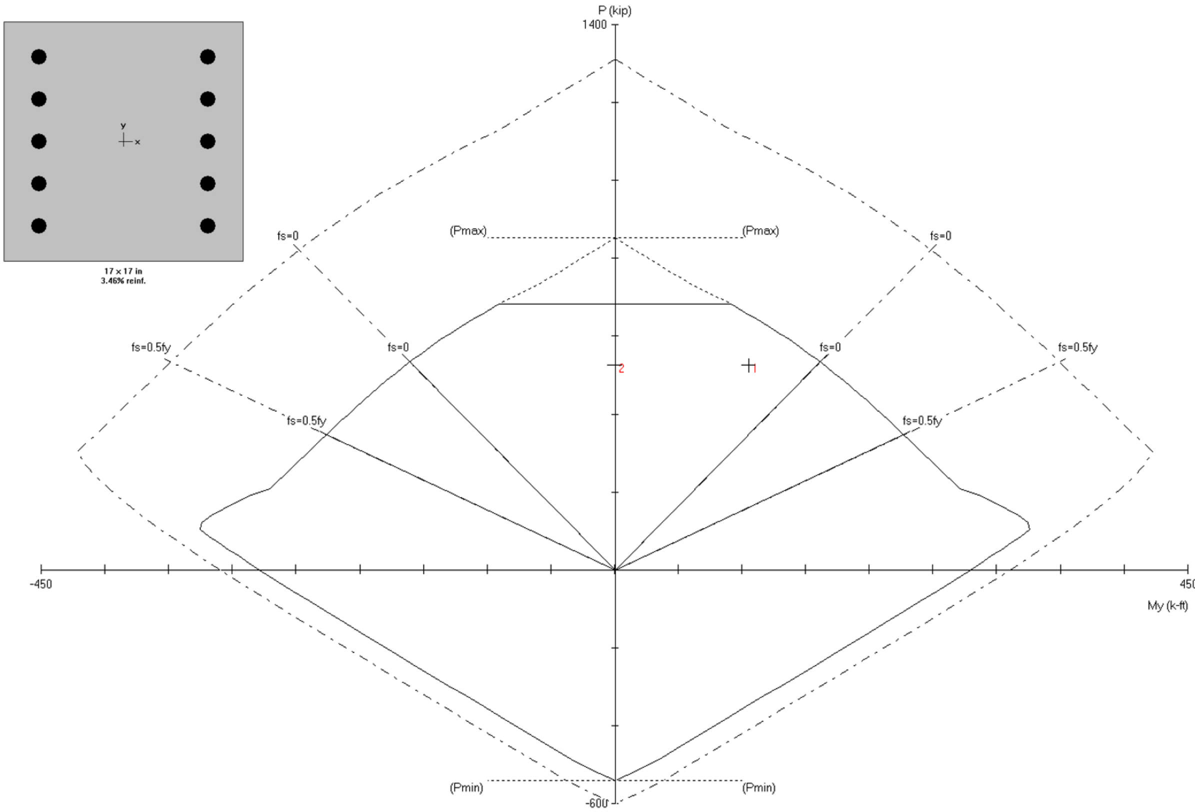


Figure 3 – Column Interaction Diagram

5. Column Design - spColumn Software

spColumn program performs the analysis of the reinforced concrete section conforming to the provisions of the Strength Design Method and Unified Design Provisions with all conditions of strength satisfying the applicable conditions of equilibrium and strain compatibility and includes slenderness effects using moment magnification method for sway and nonsway frames. For this column section, we ran in design mode with control points using the ACI 318-14. The graphical and text results are provided below for both input and output of the spColumn model.

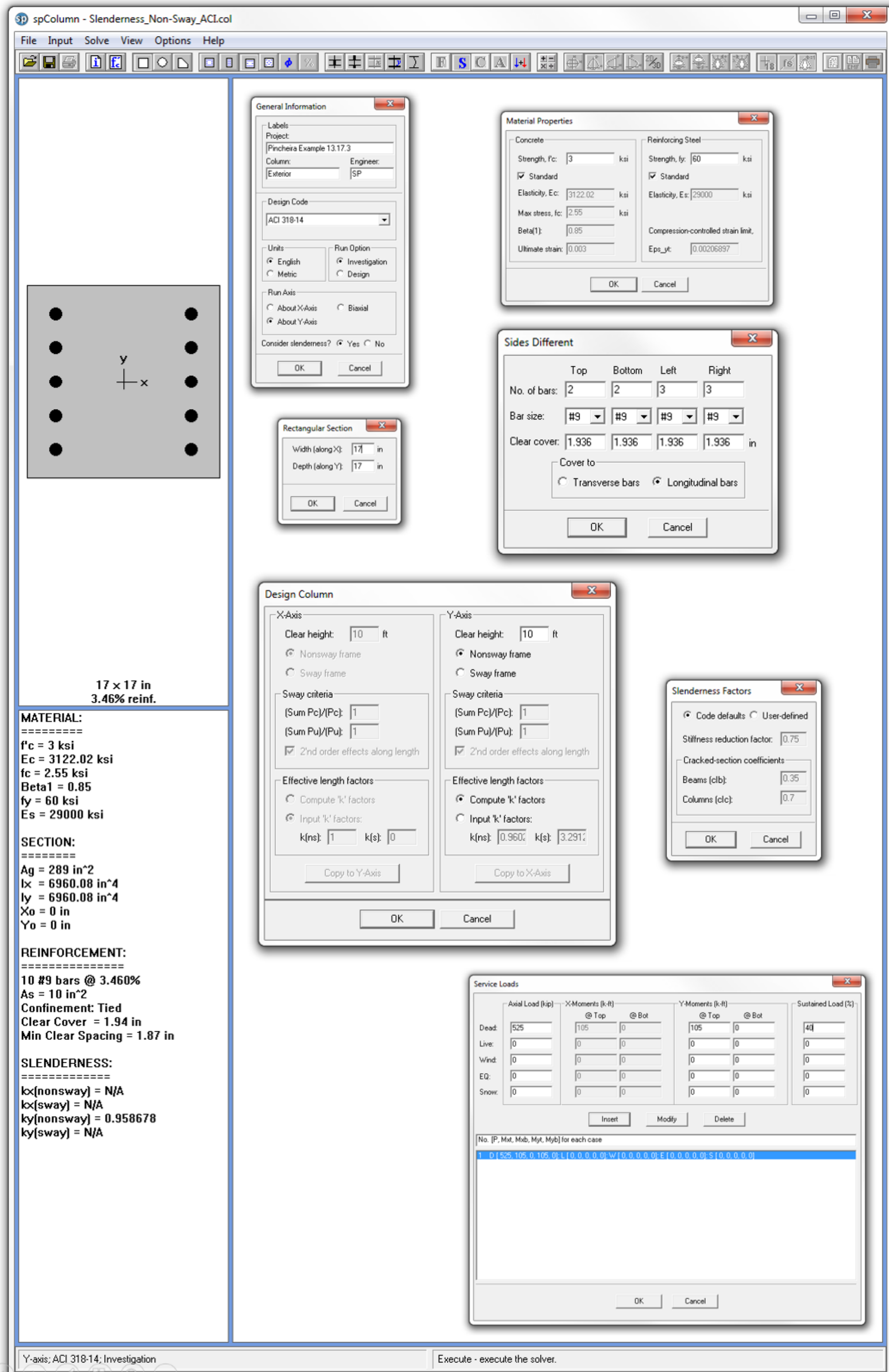


Figure 4 – spColumn Model Input Wizard Windows

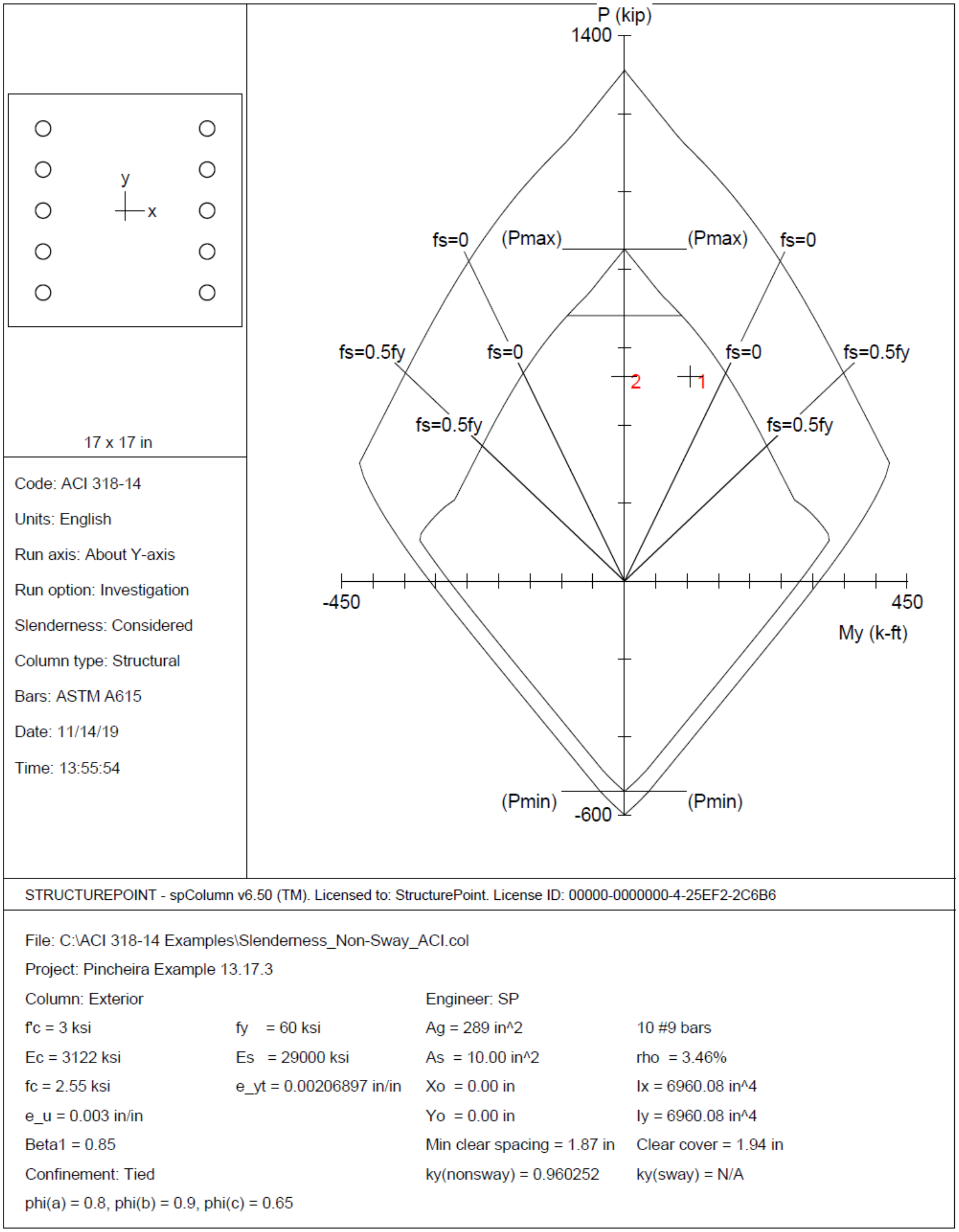
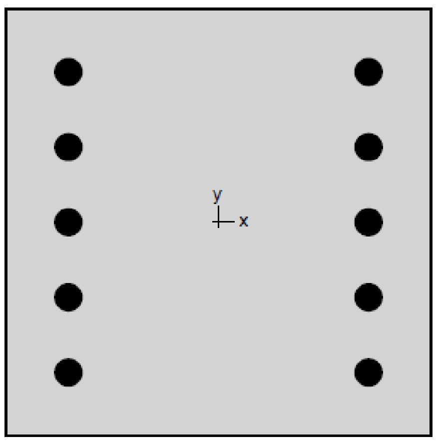


Figure 5 – Column Section Interaction Diagram about Y-Axis (spColumn)

spColumn v6.50
Computer program for the Strength Design of Reinforced Concrete Sections



Contents

List of Figures

1. General Information

File Name	C:\ACI 318-14 Exa...\Slenderness_Non-Sway_ACI.col
Project	Pincheira Example 13.17.3
Column	Exterior
Engineer	SP
Code	ACI 318-14
Bar Set	ASTM A615
Units	English
Run Option	Investigation
Run Axis	Y - axis
Slenderness	Considered
Column Type	Structural
Capacity Method	Critical capacity

2. Material Properties

2.1. Concrete

Type	Standard	
f'_c	3	ksi
E_c	3122.02	ksi
f_c	2.55	ksi
ε_u	0.003	in/in
β_1	0.85	

2.2. Steel

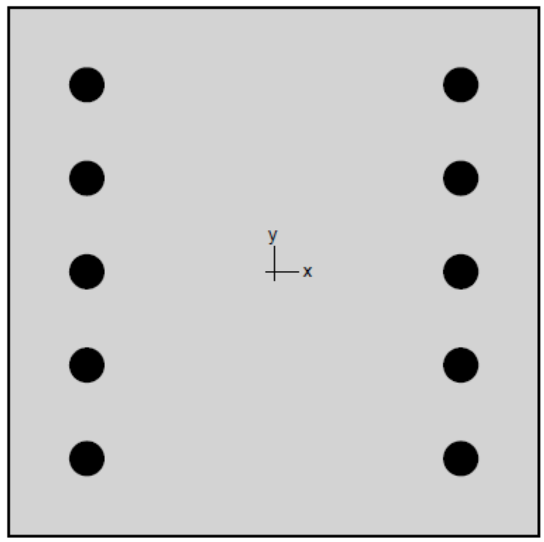
Type	Standard	
f_y	60	ksi
E_s	29000	ksi
ε_{yt}	0.00206897	in/in

3. Section

3.1. Shape and Properties

Type	Rectangular	
Width	17	in
Depth	17	in
A_g	289	in²
I_x	6960.08	in⁴
I_y	6960.08	in⁴
r_x	4.90748	in
r_y	4.90748	in
X_o	0	in
Y_o	0	in

3.2. Section Figure



Rectangular 17 x 17 in   3.46% reinf.

Figure 1: Column section

4. Reinforcement

4.1. Bar Set: ASTM A615

Bar	Diameter	Area	Bar	Diameter	Area	Bar	Diameter	Area
	in	in²		in	in²		in	in²
#3	0.38	0.11	#4	0.50	0.20	#5	0.63	0.31
#6	0.75	0.44	#7	0.88	0.60	#8	1.00	0.79
#9	1.13	1.00	#10	1.27	1.27	#11	1.41	1.56
#14	1.69	2.25	#18	2.26	4.00			

4.2. Confinement and Factors

Confinement type	Tied
For #10 bars or less	#3 ties
For larger bars	#4 ties
Capacity Reduction Factors	
Axial compression, (a)	0.8
Tension controlled ϕ, (b)	0.9
Compression controlled ϕ, (c)	0.65

4.3. Arrangement

Pattern	Sides different
Bar layout	Rectangular
Cover to	Longitudal bars
Clear cover	---
Bars	---

Total steel area, A_s	10.00 in²
Rho	3.46 %
Minimum clear spacing	1.87 in

4.4. Bars Provided

		Bars	Cover in
Top	2	#9	1.936
Bottom	2	#9	1.936
Left	3	#9	1.936
Right	3	#9	1.936

5. Loading

5.1. Load Combinations

Combination	Dead	Live	Wind	EQ	Snow
U1	1.000	0.000	0.000	0.000	0.000

5.2. Service Loads

No.	Load Case	Axial Load kip	Mx @ Top k-ft	Mx @ Bottom k-ft	My @ Top k-ft	My @ Bottom k-ft
1	Dead	525.00	105.00	0.00	105.00	0.00
1	Live	0.00	0.00	0.00	0.00	0.00
1	Wind	0.00	0.00	0.00	0.00	0.00
1	EQ	0.00	0.00	0.00	0.00	0.00
1	Snow	0.00	0.00	0.00	0.00	0.00

5.3. Sustained Load Factors

Load Case	Factor %
Dead	40
Live	0
Wind	0
EQ	0
Snow	0

6. Slenderness

6.1. Sway Criteria

Y-Axis	Non-sway column

6.2. Columns

Column	Axis	Height ft	Width in	Depth in	I in⁴	f'_c ksi	E_c ksi
Design	Y	10	17	17	6960.08	3	3122.02
Above	Y	12	17	17	6960.08	3	3122.02
Below	Y	(no column specified...)					

6.3. Y - Beams

Beam	Length ft	Width in	Depth in	I in⁴	f'_c ksi	E_c ksi
Above Left	30	14	24	16128	3	3122.02

Beam	Length	Width	Depth	I	f'c	Ec
	ft	in	in	in^4	ksi	ksi
Above Right	(no beam specified...)					
Below Left	(no beam specified...)					
Below Right	(no beam specified...)					

7. Moment Magnification

7.1. General Parameters

Factors	Code defaults
Stiffness reduction factor, ϕ_K	0.75
Cracked section coefficients, cI(beams)	0.35
Cracked section coefficients, cI(columns)	0.7
$0.2\ E_c\ I_g + E_s\ I_{se}$ (Y-axis)	1.48e+007 kip-in²
Minimum eccentricity, $e_{y\ min}$	1.11 in

7.2. Effective Length Factors

Axis	Ψ_{top}	Ψ_{bottom}	k (Nonsway)	k (Sway)	kl_u/r
Y	4.512	999.000	0.960	(N/A)	23.48

7.3. Magnification Factors: Y - axis

* Slenderness need not be considered.

Load Combo		At Ends ΣP_u	P_c	ΣP_c	β_{ds}	δ_s	Along Length P_u	$k'l_u/r$	P_c	β_{dns}	C_m	δ	
		kip	kip	kip			kip		kip				
1	U1	(N/A)	(N/A)	(N/A)	(N/A)	(N/A)	525.00	(N/A)	7850.31	0.400	(N/A)	(N/A)	*

8. Factored Moments

NOTE: Each loading combination includes the following cases:
Top - At column top
Bot - At column bottom

8.1. Y - axis

Load Combo			1st Order M_{ns}	M_s	M_u	M_{min}	2nd Order	M_I	M_c	Ratio 2nd/1st
			k-ft	k-ft	k-ft	k-ft		k-ft	k-ft	
1	U1	Top	105.00	(N/A)	105.00	(N/A)	M_2=	(N/A)	(N/A)	(N/A)
1	U1	Bot	0.00	(N/A)	0.00	(N/A)	M_1=	(N/A)	(N/A)	(N/A)

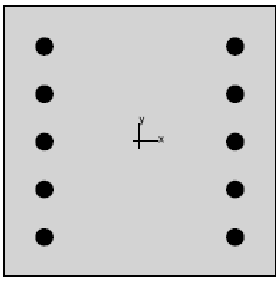
9. Factored Loads and Moments with Corresponding Capacity Ratios

NOTE: Calculations are based on "Critical Capacity" Method.
Each loading combination includes the following cases:
Top - At column top
Bot - At column bottom

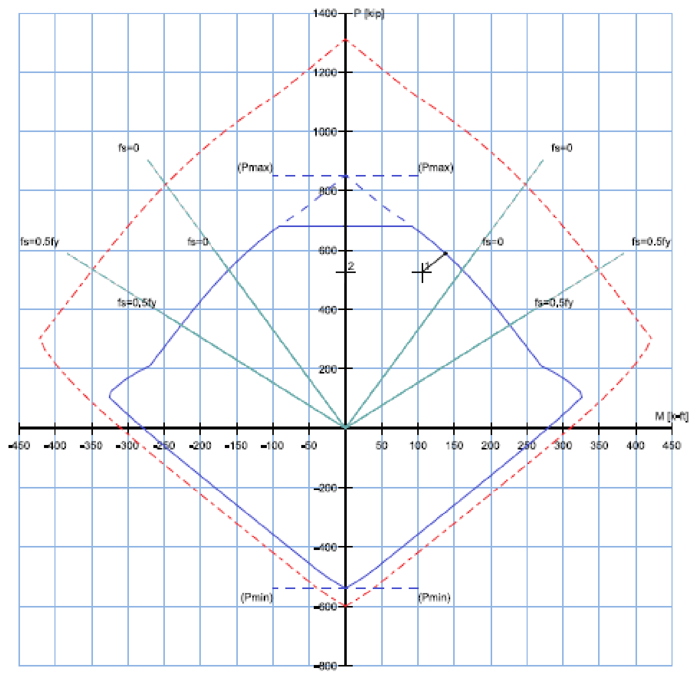
No.	Load Combo			Demand P_u	M_{uy}	Capacity ϕP_n	ϕM_{ny}	Parameters at Capacity NA Depth	ε_t	ϕ	Capacity Ratio
				kip	k-ft	kip	k-ft	in			
1	1	U1	Top	525.00	105.00	588.71	137.85	15.81	-0.00025	0.650	0.86
2	1	U1	Bot	525.00	0.00	681.95	0.00	18.48	-0.00065	0.650	0.77

10. Diagrams

10.1. PM at θ=0 [deg]



17 x 17 in



PM at 0,0 [deg]

General Information

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Column	Exterior
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Slenderness	Considered
Column Type	Structural
Capacity Method	Critical capacity

Materials

f'_c	3 ksi
E_c	3122.02 ksi
f_y	60 ksi
E_s	29000 ksi

Section

Type	Rectangular
Width	17 in
Depth	17 in
A_g	289 in²
I_x	6960.08 in⁴
I_y	6960.08 in⁴

Reinforcement

Pattern	Sides different
Bar layout	Rectangular
Cover to	Longitudal bars
Clear cover	—
Bars	—
Confinement type	Tied
Total steel area, A_s	10.00 in²
Rho	3.46 %
Min. clear spacing	1.87 in

No.	Load Combo			P_u kip	M_{uy} k-ft	ϕP_n kip	ϕM_{ny} k-ft	Capacity Ratio
1	1	U1	Top	525.0	105.0	588.71	137.85	0.86
2	1	U1	Bot	525.0	0.0	681.95	0.00	0.77

Max. Capacity Ratio: 0.86

6. Summary and Comparison of Design Results

Analysis and design results from the hand calculations above are compared with the reference values and the exact values obtained from spColumn model.

Table 2 – Parameters for moment magnification of column in non-sway frame						
	k (Note 3)	$(EI)_{eff}$, kip.in.^{2}	P_c, kip	P_u, kip	δ_{ns} (Note 4)	M_c, kip.ft
Reference	1.000	10.50×10^6	7200	525	0.66	105
Hand	0.959	10.56×10^6	7871	525	0.66	105
spColumn	0.960	10.57×10^6	7850	525	N/A	105

All the results of the hand calculations illustrated above are in precise agreement with the automated exact results obtained from the spColumn program.

The notes below are helpful to the spColumn user in creating the design model:

1. The reference used the larger of the two equations provided by ACI 318-14 (6.6.4.4.4) to calculate $(EI)_{eff}$ since both $(EI)_{eff}$ equations are lower bounds. However, the hand solution and spColumn use the first equation since it provides an estimate that is dependent on the reinforcement configuration provided in the section. 2.
2. The reference used an approximate equation to calculate the radius of gyration (r) while the hand solution and spColumn use the exact equation to calculate r value.
3. The reference decided to use k = 1 in accordance with the more conservative procedure of ACI 318-14 (6.6.4.4.3). The hand solution and spColumn calculate the exact k value.
4. δ_{ns} in the three methods of solution shown above need not be calculated since the slenderness effects need not be considered. The reference and hand solution show this value for illustration purposes.

7. Conclusions & Observations

7.1. General Observations

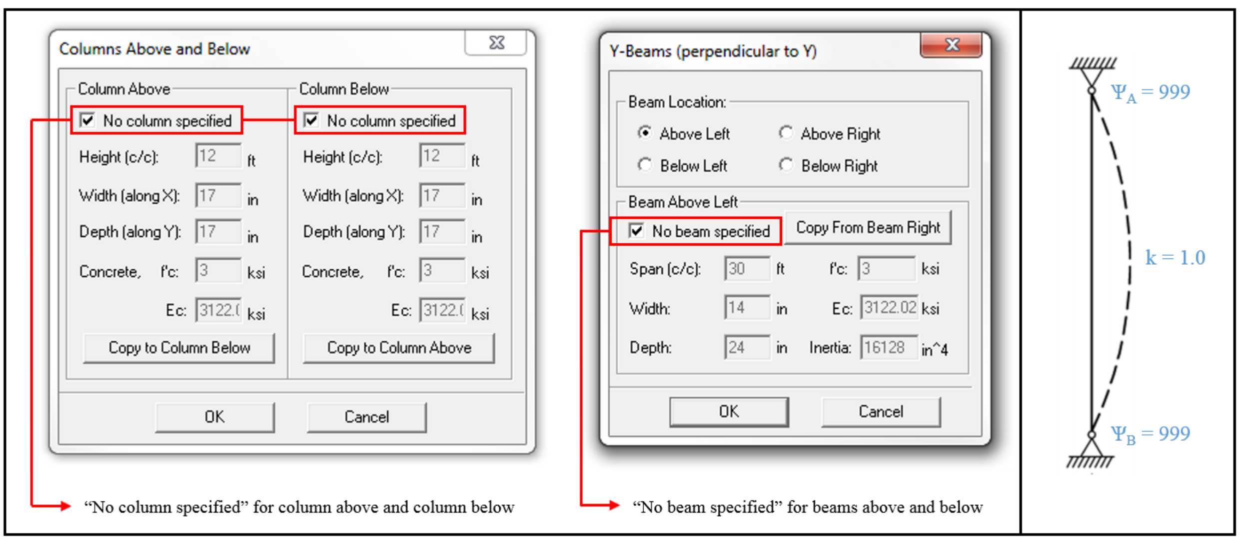
The analysis of the reinforced concrete section performed by spColumn conforms to the provisions of the Strength Design Method and Unified Design Provisions with all conditions of strength satisfying the applicable conditions of equilibrium and strain compatibility and includes slenderness effects using moment magnification method for sway and nonsway frames.

ACI 318 provides multiple options for calculating values of r and $(EI)_{eff}$ leading to variability in the determination of the adequacy of a column section. Engineers must exercise judgment in selecting suitable options to match their design condition. The spColumn program utilizes the exact methods whenever possible and allows user to override the calculated values with direct input based on their engineering judgment wherever it is permissible.

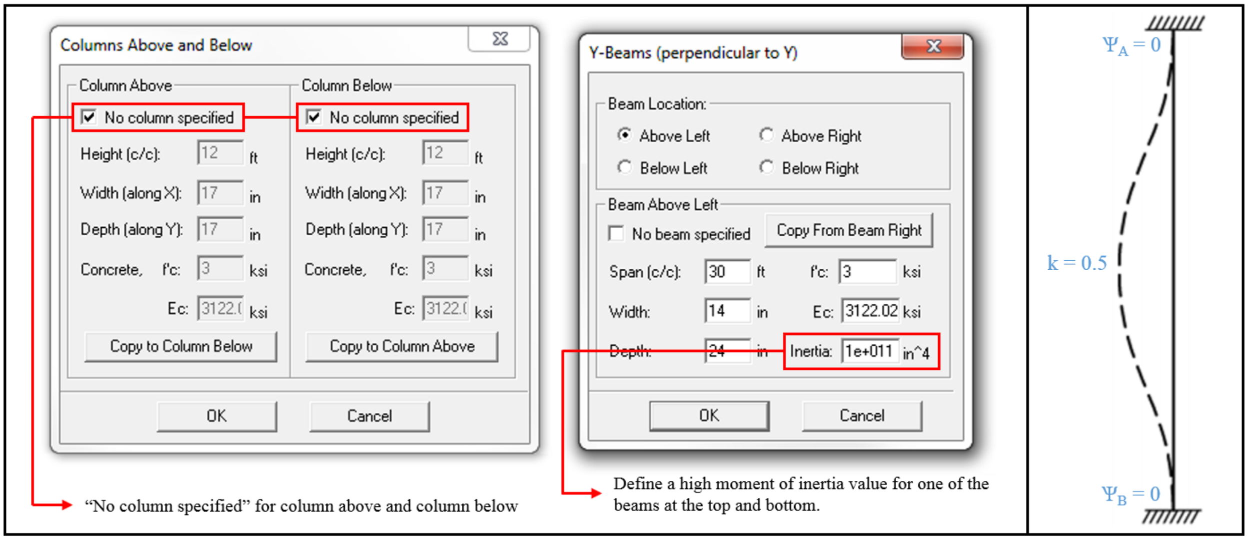
7.2. Design Column Boundary Conditions in Slenderness Calculations

When the slenderness effects for a non-sway frame column is considered in creating a model using spColumn, the effective length factor can be computed by defining the properties of the columns and beams connected to the top and bottom of the design column. The following notes are helpful when using spColumn to calculate the k value for some of the special boundary conditions cases:

1. To model pin supports at the top and bottom of the design column:



2. To model fix supports at the top and bottom of the design column:



3. To model pin support at the top and fix support at the bottom of the design column: